**Assignment – 3 - Solution**

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eMasters in Communication Systems, IITK

**EE901: Probability and Random Processes**

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**Q1:** In an experiment, a dice is rolled twice. Let X be the sum of the outcomes. Find .

**Q1 Solution:**

Let Y and Z be the outcomes of the first and second dice rolls – values shown by the dice after the roll.

The outcomes of the first roll – Y = {1,2,3,4,5,6} – 6 outcomes

The outcomes of the second roll – Z = {1,2,3,4,5,6} – 6 outcomes

Then, X = Y+Z

Total outcomes are 6\*6 = 36

The for X and the respective mapped events are:

|  |  |  |
| --- | --- | --- |
|  |  | Probability |
| (1,1) | 2 | 1/36 |
| (1,2),(2,1) | 3 | 2/36 |
| (1,3),(3,1),(2,2) | 4 | 3/36 |
| (1,4),(4,1),(2,3),(3,2) | 5 | 4/36 |
| (1,5),(5,1)(2,4),(4,2),(3,3) | 6 | 5/36 |
| (1,6),(6,1),(2,5),(5,2),(3,4),(4,3) | 7 | 6/36 |
| (2,6),(6,2),(3,5),(5,3),(4,4) | 8 | 5/36 |
| (3,6),(6,3),(4,5),(5,4) | 9 | 4/36 |
| (4,6),(6,4),(5,5) | 10 | 3/36 |
| (5,6),(6,5) | 11 | 2/36 |
| (6,6) | 12 | 1/36 |

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**Q2:** Let X be a random variable with PDF given by

1. Find the constant c
2. Find

**Q2 Solution:**

(a) To find c, we can use the PDF property

1. c=

(b) When PDF is given,

Var(X) = Second Central Moment =

1. – The expected value of

**Q3:** Let X be a Gaussian random variable with parameter (). Find the first four moments using the direct formula involving its PDF.

**Q3 Solution:**

The PDF of Gaussian random variable –

**The First Moment – Mean**

Simplifying and solving the integration by substituting

is an odd function (y) multiplied by an even function the result of which would be odd again. The integral of an odd function over a symmetric interval () will be 0

**Second Moment - Variance**

Second raw moment –

Simplifying and solving the integration by substituting

From the First Moment solution derivation, we know that:

is an odd function and its symmetric integral over the interval () will be 0

Simplifying above equation,

Defining

🡺

Now solving the main equation,

The integral – – is a gamma function of the form, with z=3/2 and

Note that the second moment of a distribution is variance, and we know variance of Gaussian distribution is But the above derivation shows the variance of Gaussian PDF as

**Why?**

**The second moment we calculated is the raw second moment of Gaussian PDF – which is with respect to 0 – .**

**is the second central moment of Gaussian distribution** – which is with respect to mean . Solving for third central moment of Gaussian distribution and substituting the value of ,

**Third Moment - Skewness**

Third raw moment –

Simplifying and solving the integration by substituting

– (1)

From the First and Second Moments solution derivation, we know that:

is an odd function and its integral over the symmetric interval () is 0

is an odd function and its integral over the symmetric interval () is 0

**Simplifying (1) for the third raw moment,**

**However, the third central moment is and solving as above would result in:**

The third moment measures the asymmetry or skewness of the distribution. Since Gaussian distribution is symmetric, its third moment is 0.

**Fourth Moment – Kurtosis (Peakedness)**

Fourth raw moment – – (2)

Fourth central moment – – (3)

From the First, Second, and Third Moments solution derivation above, we know that:

is an odd function and its integral over the symmetric interval () is 0

is an odd function and its integral over the symmetric interval () is 0

**Fourth Raw Moment =**

**Applying , for 4th Central moment 🡺**

**Q4:** Let X be an exponential random variable with parameter (. Find all of its moments using its MGF.

**Q4 Solution**

PDF of exponential RV is – ()

– Exponential RV is defined for

🡺 for

Each moment of the distribution is found by deriving derivative of MGF successively.

1. First Moment – Mean

**First Moment - Mean**

1. Second Moment – Variance

**Second Moment – Variance =**

1. Third Moment – Skewness

**Third Moment – Skewness = – As the distribution is NOT symmetric, this moment – Skewness is NOT zero**

1. Fourth Moment – Kurtosis

**Fourth Moment – Kurtosis =**

**Q5:** Let X be a uniformly distributed integer taking value between 1 and 55. Let Y = modulus(X,8). Find the PMF of Y.

**Q5 Solution:**

**PMF of X**

**Events Mapping Across X and Y**

|  |  |
| --- | --- |
| Y | X |
| 0 | {8,16,24,32,40,48} – 6 Outcomes |
| 1 | {1,9,17,25,33,41,49} - 7 Outcomes |
| 2 | {2,10,18,26,34,42,50} - 7 Outcomes |
| 3 | {3,11,19,27,35,43,51} - 7 Outcomes |
| 4 | {4,12,20,28,36,44,52} - 7 Outcomes |
| 5 | {5,13,21,29,37,45,53} - 7 Outcomes |
| 6 | {6,14,22,30,38,46,54} - 7 Outcomes |
| 7 | {7,15,23,31,39,47,55} - 7 Outcomes |
|  | Total – 55 Outcomes |

**PMF of Y**

**Q6:** Let X be a continuous random variable with uniform distribution between 0 and 1. Compute distribution of Y = 1/X in terms of PDF and CDF.

**Q6 Solution:**

🡺 🡺 ,

🡺

= g(X) 🡺 🡺

X Limits :[0,1] 🡺 Y Limits : When X=0 🡺 and X=1 🡺🡺 [1]

🡺

**PDF of Y**

**CDF of Y**

**Q7:** Let X be a continuous random variable with PDF given by if , find

**Q7 Solution:**

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When PDF of X is given, CDF of X=

**Therefore**

**Q8:** Let X be a discrete random variable with range with equal probability. Find .

**Q8 Solution:**

For a discrete RV,

**Q9:** Let X be an exponential random variable with parameter (1). Find

**Q9 Solution:**

The PDF of Exponential RV -

- does not have simple closed-form expression in terms of elementary function for this solution.

- The solution can be arrived at using either numerical methods or special integral functions.

- The special integral function that can be used for expressing this integral is “Exponential Integral function” and is written as:

**Q10:** Let X be the random variable representing the value of the number rolled of a fair 4-sided die.

(a) Write down the moment generating function for X

(b) Use this moment generating function to compute the first and second moments of X

**Q10 Solution:**

(a)

X= Roll of 4-sided fair die 🡺

The probability distribution/mass function of X is:

The MGF of X =

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(b)

First moment =

Second moment =